MATH 108: Introduction to Combinatorics, Winter 2017
HOMEWORK 1
Due Tuesday, January 24

You should solve the homework on your own. Don't use any books or the internet.

Problem 1. Find a construction of orthogonal pairs of Latin squares for $n = 4m$. Start with the following Latin square:

0	1	2	3	4	5	6	7
1	0	3	2	5	4	7	6
2	3	4	5	6	7	0	1
3	2	5	4	7	6	1	0
4	5	6	7	0	1	2	3
5	4	7	6	1	0	3	2
6	7	0	1	2	3	4	5
7	6	1	0	3	2	5	4

First, find a Latin transveral in this square. Then, modify the transversal to obtain 8 disjoint Latin transversals (which solves the case of $n = 8$).

Bonus: figure out how this construction generalizes to every $n = 4m$.

Problem 2. A *magic square* is an $n \times n$ array containing the integers $\{1, 2, \ldots, n^2\}$ such that the sum of each row and each column is the same. Prove that if there is a pair of $n \times n$ orthogonal squares, then there is also an $n \times n$ magic square.

Hint: Represent a number in $\{1, \ldots, n^2\}$ as a pair of numbers up to n.

Problem 3. A k-partitite graph is such that the vertices can be partitioned into disjoint sets $V_1, V_2, \ldots, V_k$ and there is no edge with both endpoints inside the same set V_i. If the number of vertices is $|V| = k\ell$, what is the largest possible number of edges that a k-partite graph can have? Prove your answer (don't just guess!)