MATH 113: Linear Algebra, Autumn 2018
HOMEWORK 6
Due Monday, Nov 26

Try solve the homework on your own. If you discuss with others, please list your collaborators. You can use anything that was stated in class, but don't search the internet please.

Problem 1. Let $S, T \in \mathcal{L}(V)$ such that $ST = TS$. Prove that $null(S)$ is invariant under T.

Problem 2. Let $T \in \mathcal{L}(\mathbb{C}^2)$ be defined as $T(z, w) = (8w, -2z)$. Find all eigenvalues and eigenvectors of T.

Problem 3. Suppose that $T \in \mathcal{L}(V)$ is invertible. Prove that T and T^{-1} have the same eigenvectors. How are the eigenvalues related?

Problem 4. Suppose V is a complex vector space, $T \in \mathcal{L}(V)$ and $p \in \mathcal{P}(\mathbb{C})$. Prove that the eigenvalues of $p(T)$ are exactly $p(\lambda)$ where λ is an eigenvalue of T.

Problem 5. Let $T \in \mathcal{L}(V)$ be such that $T^2 = I$ (the identity), and T has no negative real eigenvalues. Prove that $T = I$.

Bonus problem. Is there a linear operator $T \in \mathcal{L}(\mathbb{R}^n)$ such that $T^8 = I$, $T^k \neq I$ for $1 \leq k \leq 7$, and $det(T) < 0$?