CS 365 – Randomized Algorithms

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Assignment #3
(Due in class on 12/4/07. No extension!)

Problems are mostly drawn from the course text-book. Note that I will use "Problem" to refer to the problems posed at the end of a chapter, and "Exercise" to refer to the exercises contained in the text. Typically, the exercises in the text are easy and you should be able to solve them while reading the chapter.

1) [30 points] Solve Problem 4.4.
2) [30 points] Solve Problem 4.8.
3) [30 points] Solve Problem 4.9. (It might be useful to first solve Exercise 4.2 on page 75.)
4) [10 points] Solve Problem 11.1.
5) [20 points] Solve Exercise 6.1 (page 127).
6) [30 points] Solve Problem 6.16.
7) [50 points] We want to estimate a value X. Each time we measure it, we get a random value X_i, where $E[X_i] = X$, $\text{var}(X_i) = O(X^2)$, and X_i's are independent.

- Give an algorithm which outputs X' such that $|X' - X| \leq \epsilon X$ with probability at least $1 - \delta$, using $O(\frac{1}{\epsilon^2} \ln \frac{1}{\delta})$ measurements. Prove the correctness of your algorithm.
- If $var(X_i) = f(X)$, how many measurements are needed to give the same guarantee as above? Specify the answer in terms of ϵ, δ, X, and $f(X)$. You only need to provide the final answer.

Hint 1: Consider taking the average of some of the X_i's, then taking the median of the averages.

Hint 2: You may use the following *additive Chernoff bound:* If $X_1, \ldots X_n$ are n iid random variables with expectation μ, then

$$Pr[\sum X_i \geq n\mu + a] \leq e^{-2a^2/n}$$

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Reading Assignment: I assume you have finished reading Chapter 4 and the portion of Chapter 11 covered in class, i.e., Sections 11.1 and 11.2. After finishing with Chapter 6, we will be moving on to Chapter 7 — make sure you read these in the near future.