CS364A: Exercise Set #1

Due by the beginning of class on Wednesday, October 2, 2013

Instructions:

(1) Turn in your solutions to all of the following exercises directly to one of the TAs (Kostas or Okke). You can give them a hard copy or send a soft copy by email to `cs364a-aut1314-submissions@cs.stanford.edu`. Please type your solutions if possible and feel free to use the LaTeX template provided on the course home page.

(2) Your solutions will be graded on a "check/minus/zero" system, with "check" meaning satisfactory and "minus" meaning needs improvement.

(3) Solve these exercises and write up your solutions on your own. You may, however, discuss the exercises verbally at a high level with other students. You may also consult any books, papers, and Internet resources that you wish. And of course, you are encouraged to contact the course staff (in office hours or via Piazza) to clarify the questions and the course material.

(4) No late assignments will be accepted.

Lecture 1 Exercises

Exercise 1

Give at least two suggestions for how to modify the Olympic badminton tournament format to reduce or eliminate the incentive for teams to intentionally lose a match.

Exercise 2

For this exercise and the next, see Section 1.3 of the AGT book (available for free from the course Web site) for a formal definition of a Nash equilibrium.

(a) Watch the scene from *A Beautiful Mind* that purports to explain what a Nash equilibrium is. (It's easy to find on YouTube.) The scenario described is most easily modeled as a game with four players (the men), each with the same five actions (the women). Explain why the solution proposed by the John Nash character (i.e., Russell Crowe) is not a Nash equilibrium.

(b) **(Optional extra credit)** Propose a plausible game-theoretic model (probably a sequential game, rather than a single-shot game) in which the solution proposed by Nash/Crowe is a Nash equilibrium.

Exercise 3

Prove that there is a unique (mixed-strategy) Nash equilibrium in the Rock-Paper-Scissors game described in class.

Lecture 2 Exercises

Exercise 4

Compare and contrast an eBay auction with the sealed-bid second-price auction described in class. (Read up on eBay auctions if you don't already know how they work.) Should you bid differently in the two auctions?

Exercise 5

Consider a single-item auction with at least three bidders. Prove that awarding the item to the highest bidder, at a price equal to the third-highest bid, yields an auction that is *not* dominant-strategy incentive compatible (DSIC).

Exercise 6

Suppose there are k identical copies of a good and $n > k$ bidders. Suppose also that each bidder can receive at most one good. What is the analog of the second-price auction? Prove that your auction is DSIC.

Exercise 7

Suppose you want to hire a contractor to perform some task, like remodeling a house. Each contractor has a private cost for performing the task. Give an analog of the Vickrey auction in which contractors report their costs and the auction chooses a contractor and a payment. Truthful reporting should be a dominant strategy in your auction and, assuming truthful bids, your auction should select the contractor with the smallest private cost.

[Aside: auctions of this type are called *procurement* or *reverse* auctions.]

Exercise 8

Recall the sponsored search setting, in which bidder i has a valuation v_i per click. There are k slots with click-through-rates (CTRs) $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$. Recall that the social surplus of an assignment of bidders to slots is $\sum_{i=1}^{n} v_i x_i$, where x_i equals the CTR of the slot to which i is assigned (or 0 if bidder i is not assigned to any slot).

Prove that the social surplus is maximized by assigning the bidder with the jth highest valuation to the jth slot for $j = 1, 2, \ldots, k$.