CS364A: Exercise Set #8

Due by the beginning of class on Wednesday, November 20, 2013

Instructions:

(1) Turn in your solutions to all of the following exercises directly to one of the TAs (Kostas or Okke). Please type your solutions if possible and feel free to use the LaTeX template provided on the course home page. Email your solutions to `cs364a-aut1314-submissions@cs.stanford.edu`. If you prefer to hand-write your solutions, you can give it to one of the TAs in person at the start of the lecture.

(2) Your solutions will be graded on a "check/minus/zero" system, with "check" meaning satisfactory and "minus" meaning needs improvement.

(3) Solve these exercises and write up your solutions on your own. You may, however, discuss the exercises verbally at a high level with other students. You may also consult any books, papers, and Internet resources that you wish. And of course, you are encouraged to contact the course staff (via Piazza or office hours) to clarify the questions and the course material.

(4) No late assignments will be accepted.

Lecture 15 Exercises

Exercise 65

Prove that in every network cost-sharing game, the POA is at most k, where k is the number of players.

Exercise 66

Algorithmic Game Theory, Exercise 17.2.

Exercise 67

Prove that in every network cost-sharing game in which all players have a common source vertex and a common sink vertex, there is a one-to-one correspondence between strong Nash equilibria and minimum-cost outcomes. (Thus, in such games, strong Nash equilibria always exist and the POA of such equilibria is 1.)

Exercise 68

Prove that the network cost-sharing game shown in Figure 1 has no strong Nash equilibria.

Lecture 16 Exercises

Exercise 69

Exhibit a game with a pure Nash equilibrium and an initial outcome from which best-response dynamics cycles forever.

[Hint: a game with two players and three strategies per player should be plenty.]

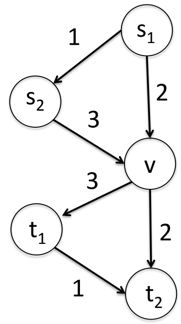

Figure 1: Network cost-sharing game for Exercise 68.

Exercise 70

Suppose a finite cost-minimization game admits a function Φ with the property that, for every outcome $\mathbf{s}$, every player i, and every deviation s'_i with $C_i(s'_i, \mathbf{s}_{-i}) < C_i(\mathbf{s})$, $\Phi(s'_i, \mathbf{s}_{-i}) < \Phi(\mathbf{s})$. Is this enough to conclude that the game has at least one PNE?

Exercise 71

Recall the result from lecture: in every atomic selfish routing game such that all k players share the same source vertex and same sink vertex, and such that $c_e(x+1) \in [c_e(x), \alpha \cdot c_e(x)]$ for every $e \in E$ and positive integer x, MaxGain ϵ-best-response-dynamics converges to an ϵ-pure Nash equilibrium in $O(\frac{k\alpha}{\epsilon} \log \frac{\phi(\mathbf{s}_0)}{\phi_{\min}})$ iterations. Here $\mathbf{s}^0$ is the initial state, $\Phi_{\min}$ is the minimum value of Φ, and in every iteration of MaxGain ϵ-best-response dynamics, among all players that have an ϵ-move (i.e., a deviation s'_i such that $C_i(s'_i, \mathbf{s}_{-i}) < (1-\epsilon)C_i(\mathbf{s})$), the one with maximum (absolute) gain $C_i(\mathbf{s}) - C_i(s'_i, \mathbf{s}_{-i})$ is chosen.

Show that the same result holds for the MaxRelativeGain variant of ϵ-best-response dynamics, where among all players with an ϵ-move, the one with maximum relative gain $[C_i(\mathbf{s}) - C_i(s'_i, \mathbf{s}_{-i})]/C_i(\mathbf{s})$ is chosen.